A measure-on-graph-valued diffusion: a particle system with collisions and their application

By

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Abstract. A diffusion taking value in probability measures on a graph with a vertex set V, $\sum_{i\in V} x_i\delta_i$, is studied. The masses on each vertices satisfy the stochastic differential equation of the form $dx_i = \sum_{j\in N(i)} \sqrt{x_i x_j} dB_{ij}$ on the simplex, where $\{B_{ij}\}$ are independent standard Brownian motions with skew symmetry and $N(i)$ is the neighbour of the vertex i. A dual Markov chain on integer partitions to the Markov semigroup associated with the diffusion is used to show that the support of an extremal stationary state of the adjoint semigroup is an independent set of the graph. We also investigate the diffusion with a linear drift, which gives a killing of the dual Markov chain on a finite integer lattice. The Markov chain is used to study the unique stationary state of the diffusion, which generalizes the Dirichlet distribution. Although no explicit expression of the density of the stationary state is available, a coupling from the past algorithm enables use of the density as a prior distribution. As an application of the latter diffusion, we discuss a Bayesian graph selection based on computation of marginal likelihoods.

Reference

Mano, S. A measure-on-graph-valued diffusion: a particle system with collisions and its applications. *Mathematics: Special Issue Random Combinatorial Structures,* 10: 4081 (2022)